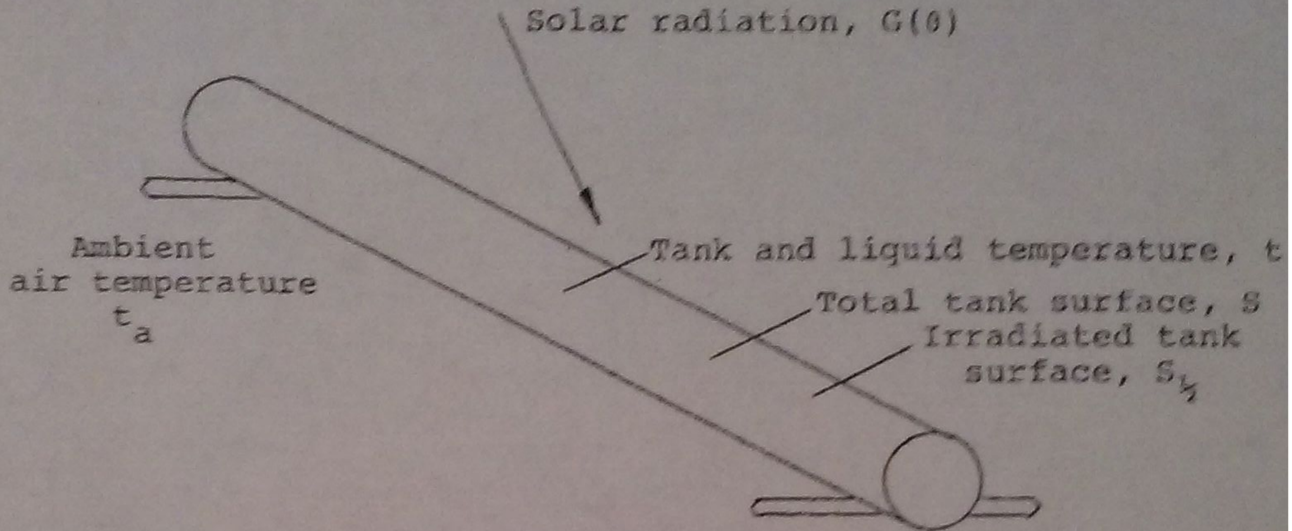

Figure 1A The Thermal System

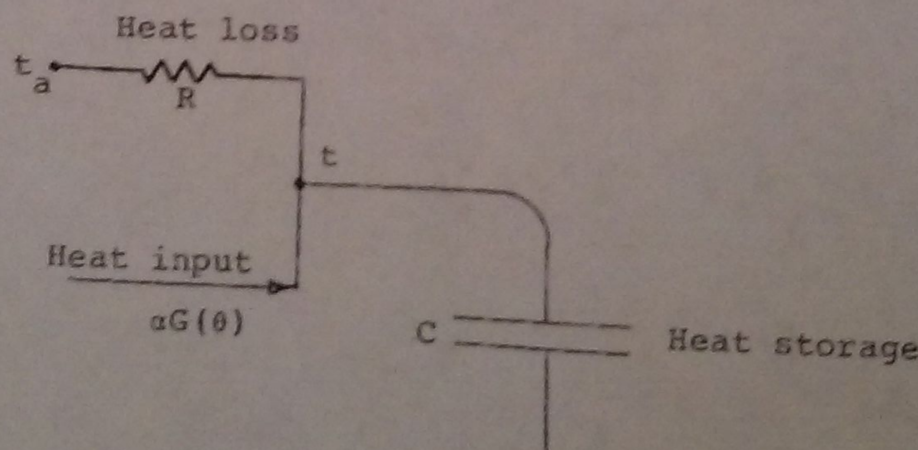

Figure 1B The Thermal Circuit

IMPORTANT SYSTEM PARAMETERS

The following system parameters are of importance.

Solar Radiation Gain

This parameter varies with time of day, time of year and the latitude. Further, cloud cover also influences the solar radiation.

Solar Reflectivity

This parameter is, in essence, the ratio of the reflected solar radiation to the incident solar radiation. Solar reflectivity is also equal to 1 - solar absorptivity for materials having no transmissivities.

Infrared Radiation Loss

Heat is lost from a warm surface to a cooler environment by infrared radiation (in terms of surface emissivity and its temperature).

Convective Heat Loss

Heat is lost from a warm surface to a cooler environment by convection (natural and forced). If there is high wind speed, the convective heat transfer can become large.

Other Pertinent Parameters

There are other system parameters that must be considered in such thermal modeling (for example, tank size and orientation).

MATHEMATICAL MODELING OF THE TANK SYSTEM*

The following postulates define an idealized thermal model that can be used to estimate the temperature level of a liquid in an uninsulated, long cylindrical tank that is exposed to solar radiation during the daytime. Heat is lost from the tank surface by convection and infrared radiation to the environment. Because this boundary value problem is a periodic type (based on reoccurring, sunny days), there is also thermal storage in the tank liquid.

A convenient, simplified solar radiation absorption function to be used in the differential equation that describes the tank liquid temperature is,

$$0.637\ \alpha G(\theta) S_{\frac{1}{2}} = 0.637\ \overline{\alpha G} S_{\frac{1}{2}} + 0.637\ \overline{\alpha G} S_{\frac{1}{2}} \sin\omega\theta \qquad (1)$$

where $\alpha G(\theta)$, absorbed solar radiation flux on a flat, perpendicular surface, Btu/hr ft^2

0.637, the shape factor for radiation falling on a cylindrical surface

$S_{\frac{1}{2}}$, one-half of the total surface of a long cylindrical tank, ft^2

$\overline{\alpha G}$, the absorbed average solar radiation flux on a flat, perpendicular surface (averaged over 24 hours), Btu/hr ft^2

ω, the period, $\pi/12$ hr^{-1}

θ, time through the cycle, hr

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*See Figures 1A and 1B.

The differential equation that describes the system heat balance is,

Heat absorbed - Heat lost = Heat stored

$$0.637\ \overline{\alpha G}S_{\frac{1}{2}} + 0.637\ \overline{\alpha G}S_{\frac{1}{2}}\ \sin\omega\theta - \frac{(t - t_a)\,S}{R} = C\,\frac{dt}{d\theta} \tag{2}$$

Rearranging the terms,

$$\frac{dt}{d\theta} + \frac{S}{CR}\,t = \frac{0.637\ \overline{\alpha G}S_{\frac{1}{2}}}{C} + \frac{t_a S}{CR} + \frac{0.637\ \overline{\alpha G}S_{\frac{1}{2}}}{C}\,\sin\omega\theta \tag{3}$$

where, t, temperature of the liquid in the tank, °F

S, the total surface area of the long cylindrical tank, ft^2

C, thermal capacity of the tank and its liquid volume (sum of the products of the weights times the specific heats), Btu/°F

R, thermal resistance of the ambient air around the tank, $\frac{1}{h_c + h_r}$, $\frac{hr\ ft^2\ °F}{Btu}$

h_c, convective heat transfer conductance for the tank, Btu/hr ft^2 °F

h_r, radiative heat transfer conductance for the tank, Btu/hr ft^2 °F

From classical differential equation theory, the temperature solution for a periodic system (after many repetitive cycles) is,

$$t = A\sin\omega\theta + B\cos\omega\theta + E \tag{4}$$

where A, B and E are constants to be determined as follows.

Substituting Equation (4) into Equation (3) yields,

$$A\omega\cos\omega\theta - B\omega\sin\omega\theta + \frac{S}{CR}\left(A\sin\omega\theta + B\cos\omega\theta\right) + E$$

$$= \frac{0.637\ \overline{\alpha G}S_{\frac{1}{2}}}{C} + \frac{S}{CR}\,t_a + \frac{0.637\ \overline{\alpha G}S_{\frac{1}{2}}}{C}\sin\omega\theta \tag{5}$$

Upon equating the cosωθ terms, one obtains,

$$A = -\frac{SB}{CR\omega} \tag{6}$$

Upon equating the sinωθ terms, and substituting Equation (6) for A, one obtains,

$$B = -\frac{0.637\ \overline{\alpha G S_{\frac{1}{2}}}}{C\left(\omega + \frac{S^2}{(CR)^2\omega}\right)} \tag{7}$$

$$\text{Thus, } A = \frac{S}{CR\omega}\,\frac{0.637\ \overline{\alpha G S_{\frac{1}{2}}}}{C\left(\omega + \frac{S^2}{(CR)^2\omega}\right)} \tag{8}$$

Upon equating the constant terms, one obtains,

$$E = \frac{0.637\ \overline{\alpha G S_{\frac{1}{2}}}\ R}{S} + t_a \tag{9}$$

The solution to this boundary value problem thus is,

$$t = \frac{S}{CR\omega}\,\frac{0.637\ \overline{\alpha G S_{\frac{1}{2}}}}{C\left(\omega + \frac{S^2}{(CR)^2\omega}\right)}\sin\omega\theta$$

$$- \frac{0.637\ \overline{\alpha G S_{\frac{1}{2}}}}{C\left(\omega + \frac{S^2}{(CR)^2\omega}\right)}\cos\omega\theta$$

$$+ \frac{0.637\ \overline{\alpha G S_{\frac{1}{2}}}\,R}{S} + t_a \tag{10}$$

INTERPRETATIONS

Small Tanks

For small tanks, the thermal capacities, C, are not large and when substituting the system parameters (S, C, R, ω and $\overline{\alpha G}$) into Equation (10), the $\sin\omega\theta$ and $\cos\omega\theta$ terms are significant relative to the last two terms in that Equation. One can use Equation (10) to determine the tank temperatures above the ambient values.

Large Tanks

When large tanks are being evaluated where the thermal capacity, C, is large, the $\sin\omega\theta$ and $\cos\omega\theta$ terms become very small and negligible; under these circumstances, Equation (10) reduces to,

$$t = \frac{0.637\ \overline{\alpha G} S_{\frac{1}{2}} R}{S} + t_{am} = \frac{0.637\ \overline{\alpha G} R}{2} + t_a \qquad (11)$$

SAMPLE CALCULATIONS FOR LARGE, LONG CYLINDRICAL TANKS

Example 1: Consider the following controlling system parameters:

$\alpha = 0.19$

$\overline{G} = 75$ Btu/hr ft^2

$h_r = 1.0$ Btu/hr ft^2 °F

$h_c = 0.5$ Btu/hr ft^2 °F (natural convection)

$$R = \frac{1}{h_r + h_c} = \frac{1}{1.0 + 0.5} = 0.667 \text{ hr ft}^2\ \text{°F/Btu}$$

$t_a = 75$ °F

Therefore,

$$t = \frac{0.637\ \alpha\ \overline{GR}}{2} + t_a = \frac{0.637(0.19)(75)(0.667)}{2} + 75$$

$$= 3.03 + 75 = 78.03\ °F$$

Example No. 2

$\alpha = 0.75$ (ordinary, nonreflective paint)

$\bar{G} = 75$ Btu/hr ft^2

$h_r = 1.0$ Btu/hr ft^2 °F

$h_c = 0.5$ Btu/hr ft^2 °F

$R = 0.667$ hr ft^2 °F/Btu

$t_a = 75$ °F

Therefore,

$$t = \frac{0.637\ \overline{\alpha GR}}{2} + t_a = \frac{0.637(0.75)(75)0.667}{2} + 75$$

$$= 11.95 + 75 = 86.95\ °F$$

Example No. 3

Same input parameters as Example 1, except that $h_c = 7.0$ (for a 20 mph wind).

$t = 0.57 + 75 = 75.57$ °F

Example No. 4

Same input parameters as Example 2, except that $h_c = 7.0$ (for a 20 mph wind).

$t = 2.24 + 75 = 77.24$ °F